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Learning by the Sun: Observing Seasonal Declination With a Vertical Sundial

Judy L. Klein[1,3] and Adrian Riskin[2]

We designed and constructed a sundial for the purpose of observing the declination of the sun and thus marking solar seasonal variation. The 122 × 122 cm vertical sundial on the south-facing wall of our library has two unusual features: a nodus on the gnomon that casts a shadow of a point for marking the height of the sun and a large blank working space for students to mark the shadow of the nodus at different hours of the day and to connect the marks of 1 day in a line of declination. We discuss the design of a dial that emphasizes a working space for observations on solar declination, methods for determining the position of the nodus such that lines of declination can be observed every day of the year, mathematical procedures that minimize error in laying out the hour lines for this unusual design, the type of materials that are best suited for this design, and how students in an interdisciplinary honors colloquium on seasonal rhythms used the sundial. We also include two appendices on general sundial construction that indicates the information designers need to construct their own sundials.

KEY WORDS: lines of declination; seasonal variation; solar declination; sundial.

We designed and constructed a sundial for the purpose of observing the declination of the sun and thus marking solar seasonal variation. The 122 × 122 cm vertical sundial on the south-facing wall of our library has two unusual features: a nodus on the gnomon that casts a shadow of a point for marking the height of the sun at all times during the years, including the extreme solstices, and a large blank working space for students to mark the shadow of the nodus at different hours of the day and to connect the marks of 1 day in a line of declination. In addition to the mathematical challenges of designing such a working space, we faced decisions over what materials would enable us to preserve the marked lines of declination over a semester, but then erase them for another class to begin observation anew. These special needs took us beyond published instructions for constructing sundials as garden ornaments, and we hope that our summary here will enable other teachers interested in constructing pedagogical sundials to benefit from our trial and error pursuits. In the sections that follow we discuss the design of a dial that emphasizes a working space for observations on solar declination, methods for determining the position of the nodus such that lines of declination can be observed every day of the year, mathematical procedures that minimize error in laying out the hour lines for this unusual design, the type of materials that are best suited for this design, and how students in an interdisciplinary honors colloquium on seasonal rhythms used the sundial. We also include an appendix on general sundial construction that indicates the information designers need to construct their own sundials and sources on the Internet for finding that information for specific localities.

DESIGN FOR OBSERVING LINES OF DECLINATION

Sundials tell us the time of year as well as the time of day and we can use dial-face marks of our

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observations over time to map the key features of a spatial representation of the earth, including the equator and the tropics of Cancer and Capricorn. Almost all sundials mark a noon line, most have lines marking other hours, and many of the older European sundials also have lines of solar declination carved or painted on the dial face. These lines indicate the angle of declination of the sun (of the ecliptic plane of the earth's yearly orbit around the sun) with the earth's equator (the equatorial plane determined by the earth's daily rotation). For example, at the equinox, the angle of declination would be 0° and at the June solstice, the angle of declination would be 23.26° north of the equator. Typical lines of declination permanently painted on older sundials include those indicating the solstices, equinoxes, and entry points into zodiac signs. Books on sundial construction often give instructions on how to mathematically deduce and permanently mark key lines of declination. Waugh (1973, p. 138) even suggests constructing sundials that have a line of solar declination for the recipient's birthday or anniversary. We, however, wanted to work from induction—we wanted students to daily observe the altitude of the sun and to comprehend how that altitude, for a given time, changed over the course of the semester. We took inspiration from an eighteenth century vertical dial in Hesketh Park in Dartford, England (figure in Daniel, 1980, p. 10). The dial face had permanent lines of declination, but we were impressed with the fact that those lines were prominent and the hour lines were only around the periphery of the dial face.

As you can see from the figures of our dial, the hour lines are only in the 10-cm edge that borders the large working space for marking the lines of declination. Students learned how to determine the position of the hour lines on a dial face for any given latitude, and why the time of day given by our sundial usually differed from clock time (see Appendix B). Their main task in the course on seasonal rhythms, however, was to understand how the azimuth of the sun (angle along the horizon, with zero degrees corresponding to north) changed over the course of day and how the altitude of the sun for a particular time of day (angle up from the horizon, so angle of solar declination plus latitude of location) changed over the course of a season. Each student was assigned a week for observation and during that week she was expected to on at least one sunny day mark several times during the day the position of the shadow of the nodus on the dial face. At the end of the day she would use a draftsman's flexible curve to connect her penciled points in one curve of declination. Each week the entire class would go out to the sundial to reflect on the seasonal story emerging from the changing position and shape of the lines of declination. Obviously, determining the form and position of the nodus was an important part of our design process. With the help of a local metal crafter who made our gnomon and who had constructed his own equatorial sundial, we designed a nodus that took the form of a well-defined narrow point of the gnomon in the middle of a circle.

One problem we faced in designing the face of the dial to allow student observation of the lines of declination lay in deciding how far out along the gnomon to place the nodus so that the lines of declination in the most extreme cases—the solstices—would fall on the blank area on the face of the dial for at least a few hours around noon. It would also have been possible to design the size of the dial to fit a predetermined nodus location but this turned out to be less convenient given various constraints such as available funding and space. We found it easiest to determine the nodus location experimentally, using the formula

$$L^2 = \ell^2 \frac{\tan^2 \psi + \sin^2 \beta}{\cos^2 \beta} \tag{1}$$

where L is the length of the shadow cast by that portion of the gnomon between its base (its foot) and the nodus, ℓ is the height of the nodus above the dial face, ψ is the sun's azimuth, and β is the sun's angle of elevation. The derivation of this formula is elementary but complex and might make a good exercise for motivated students.

For example, if the dial is to be located at a latitude of 32.45°, the colatitude (see Appendix B for why colatitude is used in vertical dials) and hence the angle that the gnomon makes with the face is $\varphi = 90^\circ - 32.45^\circ = 57.55^\circ$. Suppose that the nodus is located 10 cm along the gnomon from its foot. Then $\ell = 10 \sin 57.55^\circ = 8.4386$ cm. At 3 P.M. on July 23, 1999, the sun's angle of elevation was $\beta = 51.6^\circ$ and the azimuth was $\psi = 253.2^\circ$. Hence $L^2 = (8.4386)^2\,[(\tan^2 253.2 + \sin^2 51.6)/\cos^2 51.6] = 2138.12$, so that $L = 46.24$ cm. This means that on this day at this time the length of the shadow of the part of the gnomon between its foot and the nodus is 46.24 cm along the 3 P.M. hour line. In order to make sure that our dial's face would accommodate the marks that we wanted our students to make, we adjusted the location of the nodus so that (a) on the summer solstice between 10 A.M. and 2 P.M. its shadow would fall on the blank area of the face and (b) on the winter solstice its shadow would remain clear of the plate through which the gnomon was bolted to the dial's face (source of data for azimuth and altitude for any time and any location is given in Appendix A).

A METHOD OF ENHANCING ACCURACY OF HOUR LINE POSITIONS

Another problem we faced during the construction of the dial was in laying out the hour lines accurately. The noon line vertically bisects the face of the dial and then the angles that the other hour lines make with the noon line can be calculated via the formula

$$\theta = \arctan(\sin\varphi \times \tan(d \times 15)) \qquad (2)$$

where φ is the colatitude of the location of the dial and d is the number of hours between noon and the hour in question. For instance, if the dial is to be located at a latitude of 32.45°, the colatitude is $\varphi = 57.55°$, so that the angle that the 2 o'clock line makes with the noon line on a south-facing vertical dial is $\theta = \arctan(\sin 57.55 \times \tan(2 \times 15)) = 25.9755°$.

The problem arises from the fact that not only is it difficult to measure angles with any accuracy of which to speak, but when lines are extended the errors in the angle measurements are multiplied to the extreme. We solved this problem by using linear measurements along the edges of the dial face rather than angle measurements. For instance, if we're making a dial with a face of 1 m^2, with the foot of the gnomon located 10 cm from the top edge of the dial, and with the hour lines confined to a 10 cm band around the edge (see Fig. 1) then $\theta = 25.9755°$, $|GA| = 80$ cm, and $|GC| = 90$ cm. We can then calculate $|AB|$ and $|CD|$ using elementary trigonometry: $|AB| = 80 \tan\theta = 38.9763$ cm and similarly $|CD| = 43.8483$ cm. With these distances in hand the hour line segment $\overline{BD}$ can be easily and accurately laid out.

For hours before 10 A.M. and after 2 P.M. the hour lines intersect the vertical sides of the dial face rather than the horizontal bottom side, and so one must adjust measurements accordingly. For instance at colatitude 57.55° the hour line for 8 A.M. makes an angle $\theta = \arctan(\sin 57.55° \times \tan 60°) = 55.62°$ with the noon line. Using the same dimensions as in the previous example (see Fig. 2) we can calculate as follows: $|GA| = 40$ cm, $|GC| = 50$ cm, and $\varphi = 90° - \theta = 34.38°$. Hence $|FB| = |FA| + |AB| = 10 + 40 \tan 34.38° = 37.3681$ cm. The distance $|ED|$ can be calculated similarly, and with these distances known the hour line segment $\overline{BD}$ can be easily and accurately laid out as well.

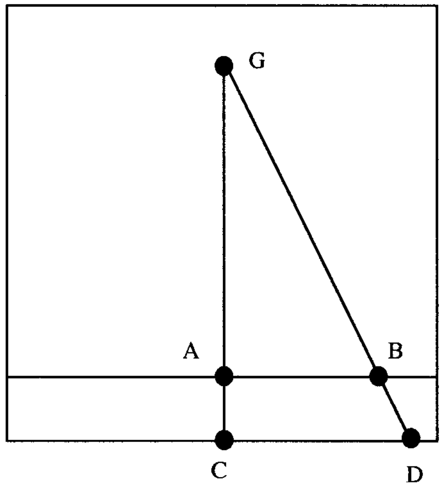

Fig. 1.

CONSTRUCTION AND INSTALLATION OF THE SUNDIAL

We made the face of our dial out of enameled aluminum signboard, which is readily available from any sign maker, who can also cut it to size. This turned out to be the best material for our purposes because it is weatherproof, durable, and can be easily marked on with a very soft, erasable artist's pencil. One disadvantage of this signboard is that it is glossy, so there is a reflection of the gnomon in addition to the proper shadow and for students working on the dial face sunglasses were a necessity. We had the signboard

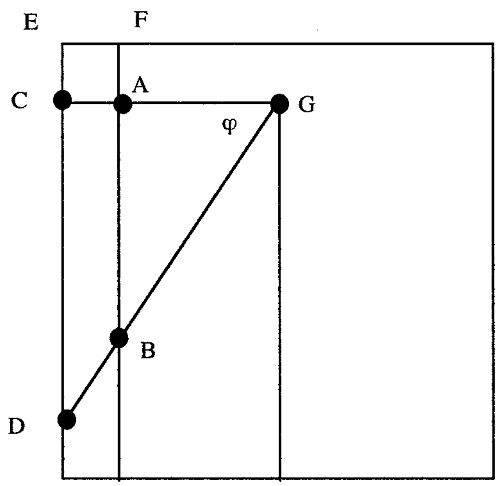

Fig. 2.

mounted on a wood back and framed to stop water warping the structure. We laid out the hour lines with car striping tape obtained from an auto parts store, and for numbers we used waterproof sign decals. These we obtained from our college's physical plant but they're also available from signmakers or in hardware stores for mounting on mailboxes.

The day of reckoning involved more reckoning that we had imagined. The recollection of the director of our physical plant and our crude compass readings, taking into consideration magnetic declination, indicated that the wall of the library was facing due south. When several members of the college's physical plant team mounted the sundial on the wall, we immediately realized that it was indicating the wrong time. Measurements of the angle of the gnomon, and a check on the position of the hour lines indicated that the fault lay not in our calculations or construction but in our assumption that the wall faced directly south (the compass readings were inaccurate due to nearby metal structures). It was a discouraging day, and the sundial lay unmounted for several more days while we wrestled with a correction. We essentially had to learn the lesson we had designed the sundial for—the potency of observation. Using observations on the shadow of a nail perpendicular to the wall and US Navy estimates of the azimuth and altitude of the sun at specific times at our location, we measured the declination of the wall—a step we should have started with and one that is explained in several texts including Waugh (1973, pp. 77–88). As a check on that measurement we carefully observed and measured shadows of vertical gnomons we had erected on the library terrace at noon local apparent time. We concluded that the wall faced 7° east of celestial south, and we had to mount the dial at that angle which meant that the eastern edge of the dial had to be 15.5 cm out from the wall. Once the correction was made and the sundial mounted, it proved very accurate in telling the time of year and the time of day (taking into consideration the reconciliation with watch-time needed due to longitude correction, daylight savings time, and the equation of time, see Appendix B) (Figs. 3–5).

USING THE SUNDIAL IN AN INTERDISCIPLINARY COURSE

The first classroom use of the sundial was in Judy's honors colloquium on the Economics, Science and Literature of Seasonal Rhythms. The major goal of this course was to explore interactions of nature and culture manifest in seasonal patterns. We examined those interactions by working our way through the layers of solar energy, meteorological phenomena, biological rhythms, agricultural production, industrial activity, monetary flows, and government policy adjustments. In addition to the empirical and

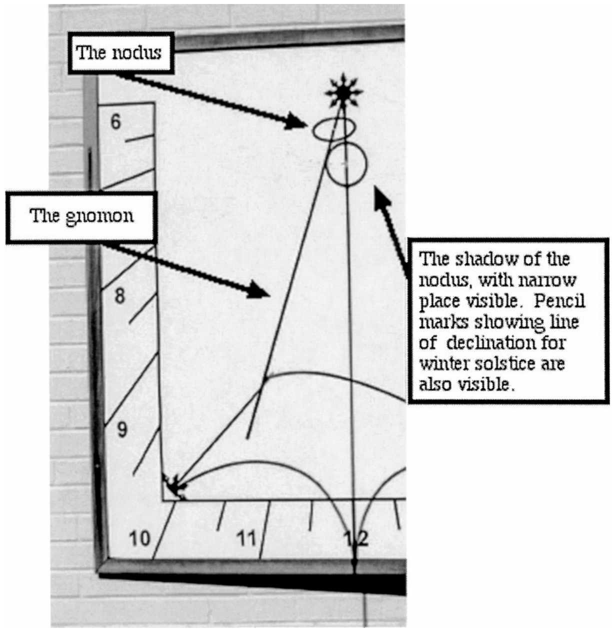

Fig. 3. The sundial on the winter solstice at noon local apparent time or 12:14 P.M. conventional time.

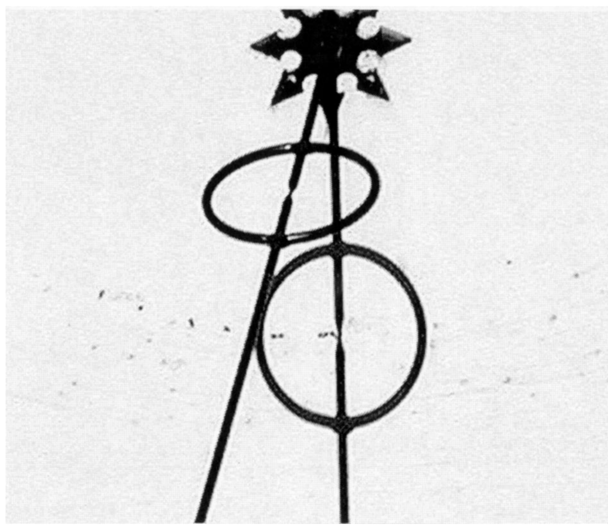

Fig. 4. Close-up of the nodus (upper circle) and its shadow (lower circle) on the winter solstice. The dark pencil marks are from observations recorded at earlier times that solstice morning. The lighter marks connected by faint lines of declination were recorded on previous days that autumn.

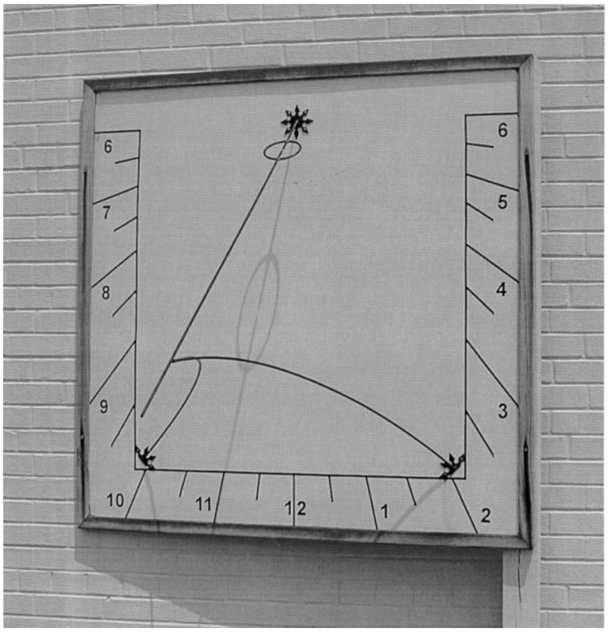

Fig. 5. Full view of dial face on July 22 at 11 A.M. local apparent time or 12:23 P.M. conventional time.

mathematical work on the sundial, (Ottewell 1979, René 1996, Waugh 1973) we used history (e.g., Aveni, 1997; Borst, 1993; Evans, 1998; Kussmaul, 1993; Lamb, 1982; Le Goff, 1980; Malville and Cluadia, 1993; Nissenbaum, 1996) and literature (e.g., the poetry of Hesiod, Thomas Hardy, and John Keats) to grasp how earlier cultures marked and remarked on seasonal variation. We also studied how the seasonal patterns of production, commerce, and financial crises in European and North American economies have changed over the past two centuries (see, e.g., Jevons, 1884; Kemmerer, 1910; Klein, 1995). Our weekly observations on solar declination reminded us of the original stimulus for all seasonal variation.

Although the students were not involved in the construction of the college sundial, they were responsible for understanding how to design dials and interpret the shadows. Examples of questions pertaining to sundials in take-home assignments include

1. Choose one location for which you will design your own sundial (either horizontal or vertical direct south dial). Fill in the blanks of the first page of the sundial construction sheet [see Appendix A]. Figure out the angle of the gnomon and the hour angles for your dial.
2. In our class we are plotting out lines of declination by observing the changing position on the dial face of the shadow of the nodus on the gnomon. The usual practice, however, is to draw the lines of declination for the equinoxes and solstices in the design and construction phase. Your job is to calculate where the shadow of our nodus should have fallen at local noon on the fall equinox and to see how far off our observation was from this position. You will need the following information to make that calculation . . ..
3. What is the remarkable feature for our colatitude of the magnitude of the altitude of the sun at noon local apparent time on the fall equinox?
4. What is unusual about the risings and settings of the sun on the equinox that can help explain why the line of declination for the equinox and only the equinox forms a straight line?
5. The MBC vertical direct south sundial, with the given angle of our gnomon, could work with some, adjustment in the dial face, in certain locations in the southern hemisphere as a vertical direct north sundial. Name a country in the southern hemisphere where we could use our same gnomon for an accurate vertical direct north sundial. Explain your choice. Assuming that at this ideal location we use the MBC gnomon, orientate the dial due north, design the dial face with the same dimensions as ours (122 × 122 cm), and we include permanent lines of declination for the solstices and the equinoxes, explain how we would have to change, if at all, any of the following to adapt to our new location: Position of the Noon line; angles of the hour lines from the noon line; counterclockwise order of numbering of hour lines; position of the equinox line of declination; and position of the December solstice line of declination.

For a fall equinox dedication of the sundial, the students wrote a pamphlet and made oral presentations on the history of sundials and how they work. The sundial was dedicated to the learning experience that comes from contemplating the relationship between our earth and the sun. Many students, faculty, and staff attended the dedication and were moved by how our attention to the sun on the equinox brought us closer to other cultures who have attempted to capture the sun's unique movement on this day—the only day of the year, along with the spring equinox, when the sun rises due east and sets due west and when everyplace in the world experiences the same amount of daylight while simultaneously undergoing the greatest daily change in light. On that day, and only on the equinox, the connected pencil marks of the shadow of the nodus formed a straight line.

By keeping track of changes in the sun's angle of declination over the course of a fall semester, students comprehended the nature of the dynamic relationship between the earth's equatorial plane and the ecliptic plane and the use of projective geometry to capture this temporal three-dimensional relationship in a static two-dimensional picture. We also gained an appreciation for the sun and for astronomical monuments and instruments used by other cultures to carefully observe and figure out the seasonal relationship between the sun and the earth.

APPENDIX A

What You Need to Know About Your Location Before Constructing and Interpreting a Vertical Sundial

—	Latitude	Latitude determines almost every feature of the sundial including the angle of the gnomon to the dial face and the angle of the hour lines from the noon line. Use a USGS map, Global Positioning System (GPS) device, or the following web page to identify your latitude and longitude http://mapping.usgs.gov/www/gnis/gnisform.html
—	Colatitude	This is 90° minus your latitude. Vertical direct dials are 90° transformations of a horizontal dial, so on vertical dials the angle of the gnomon from the dial face is equal to the colatitude, which is also used in calculating the angles of the hour lines from the noon line.
—	Longitude	You will use your longitude to reconcile sundial time and watch time. Use a USGS map, GPS device, or http://mapping.usgs.gov/www/gnis/gnisform.html
—	Magnetic declination	The gnomon for the sundial must be exactly aligned with the exact north south line. If you are using a compass to determine exact south, then you must adjust the compass reading for the magnetic declination of your location. You can get this from the US National Geophysical Data Center web page: http://www.ngdc.noaa.gov/cgi-bin/seg/gmag/fldsnth1.pl
15°	Hour angle for sun	Relative to the equatorial plane of the earth, the sun moves 15° in 1 h (i.e., it travels 1° every 4 min). This value is needed for the calculation of the hour angles and also to determine longitudinal correction needed to reconcile sundial time and watch time.
—	Number of your time zone	This is needed to get information from the US Naval Observatory on the sun's altitude and azimuth to determine lines of declination for certain days in the Eastern Time Zone. For example, the Eastern Time Zone in the United States is minus 5 (5 h slower than GMT or UT).
—	Standard meridian for your time zone	The standard time for each zone only coincides with local apparent time at one meridian in that zone. For example in the US eastern zone, the time is for 75°W longitude ($15° \times 5 = 75°$).
—	Longitudinal correction	This is the difference, in your longitude and the standard meridian for your time zone, multiplied by 4 min for each degree difference. For example, the longitude in Staunton, VA, is 79° but the time meridian for the eastern time zone is 75°, so sundial time in Staunton is 16 min slower than watch time even without the additional corrections for equation of time and daylight savings time.

Other Data Needed But Subject to Daily Changes in Value

Correction for Equation of Time. You need this to reconcile sundial time and mechanical watch time. The value of the discrepancy varies daily in an annual cycle (analemma) due to the earth's tilt relative to the sun and its varying speed during its elliptical orbit. The range spans from the dial being 16 min fast on November 11 to 14 min slow on February 5 to no discrepancy on December 25. The same values for the equation of time apply to every location. You can therefore get this from a table in almost any book on constructing sundials, or from the Royal Observatory at Greenwich web page: http://web006.pavilion.net/users/aghelyar/sundat.htm

Sun's Declination or Azimuth and Altitude. This is needed if you want to determine through mathematics, rather than observation, the seasonal lines of solar declination (solstices, equinoxes, months, zodiac signs, or anniversary lines) on your dial face. These lines of declination depend on the daily value of the sun's declination to the equator and on your latitude. The angle of the sun's declination is tabulated in most sundial books, but you can shorten your calculations by finding the altitude and azimuth for each hour per to your location on a specific day from the US Naval Observatory: http://aa.usno.navy.mil/aa/data/

APPENDIX B

Designing, Installing, and Using a Vertical Direct South Dial in the Northern Hemisphere

Designing the Gnomon

The most important property of the gnomon is the angle that the shadow-casting edge (the upper edge of gnomon, called the style) makes with the dial face. For horizontal dials, this angle would be equal to the latitude of the location of the sundial. A vertical direct south dial is a 90° transformation of the horizontal dial, so you make the angle of the gnomon equal to the colatitude for your location. The point where the style and the dial face intersect is called the foot of the gnomon. All hour lines on the dial face emanate from the foot. On a vertical dial, the foot is at the top of the dial face. The length of the gnomon varies according to desired size of dial face.

Drawing the Noon Line

Draw a meridian line through the middle of the dial face that will serve as the noon line. The noon line is the most important hour reference mark of the dial face. When the sundial is correctly positioned, the noon line will be exactly perpendicular to the north-south meridian of that location and the plane of the gnomon will be exactly perpendicular to the noon line. The foot of the gnomon will be at the top end on the noon line and the angles of all other hour lines will be measured from the point where the foot of the gnomon and the noon line intersect. If your gnomon is thick then you must draw two parallel noon lines separated by the magnitude of that thickness.

Drawing the Other Hour Lines on the Dial Face

The usual procedure is to calculate the angle of each hour line from the noon line (where it intersects with the foot of the gnomon). Relative to the equatorial plane of the earth, the sun moves 15°/h or 1° every 4 min, so the equation for determining each hour angle on dial face is

$$\tan(\theta) = \sin(\phi) \times \tan(d \times 15^\circ)$$

where θ = angle between hour line and noon line, ϕ = colatitude (for designing a horizontal dial one would use the latitude), and d = number of hours difference between noon and hour in question. Instructions are given in the text of the article on how to draw hour lines using linear measurements from the edge of the dial face.

For vertical dials, the morning hours' shadows will fall to west of the noon line and afternoon shadows will fall to east or right side of dial face so the hour lines are numbered counterclockwise. For every latitude, 6 A.M. will be 90° to the west of the noon line and 6 P.M. will be 90° to the east of the noon line.

Positioning Sundial

The vertical direct south dial is a dial face on a wall that is facing due south. Determine the declination of the wall (see, e.g., Waugh, 1973, pp. 77–88), and use a wedge if necessary so that the dial is facing due celestial south (or magnetic south plus or minus the degrees of magnetic declination for your location). The plane of the gnomon, aligned with the noon line, should be perpendicular to the vertical surface and perpendicular to the level ground. For locations north of the Tropic of Cancer, the sun will always be in the southern half of the sky so the shadow cast by the gnomon will fall on the dial face during most of the day. Books such as Waugh (1973) give guidelines for constructing vertical dials on walls facing in other directions. Vertical dials not facing directly south, north, east, or west, are called vertical declining dials.

Telling the Time of Day

The shadow of the edge of the gnomon on the dial face will tell you the local apparent time—for example, if the shadow falls on the noon line, then at that moment in your location the sun is due south (azimuth = 180° East of North) and at its highest altitude for that day. It may not, however, be noon by your watch. You need to make two or three adjustments to obtain digital watch time from a sundial reading of local apparent time.

1. *Adjusting for daylight savings time.* For most of the United States from the first Sunday in

April until the last Sunday in October, you need to add 1 h to sundial time.

2. *Adjusting for longitude.* For every 1° your longitude west of Greenwich exceeds the standard meridian of your time zone, add 4 min to sundial time. For every 1° your longitude is less than the standard meridian, subtract 4 min.
3. *Adjusting for the tilt and elliptical orbit of earth with equation of time.* Add or subtract minutes according to your reading from a table of graph for the equation of time. On 4 days of the year, April 15, June 14, September 2, and December 25, there is zero correction to be made due to the equation of time.

Telling the Time of Year

The length of the shadow of the gnomon or the position of the shadow of a nodus on a dial face informs the viewer of the time of the year. The seasonal markings—for example, lines that inform the viewer that it is the day of a solstice—are called lines of declination since they are ultimately derived from the angle of declination of the ecliptic plane (the plane in which the earth revolves around the sun) from the earth's equatorial plane. You can deduce mathematically the location on the dial face of lines of declination by finding out the altitude and azimuth of the sun for your location on the day in question for each hour. You can obtain values for the sun's altitude from the US Naval Observatory web page: http://aa.usno.navy.mil/aa/data/

The full equation is given in the text, but at noon local apparent time, the tangent of the altitude multiplied by the perpendicular height of the nodus will give you the length of the shadow from the foot of the nodus (as if a perpendicular line were dropped from the nodus to the dial face).

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